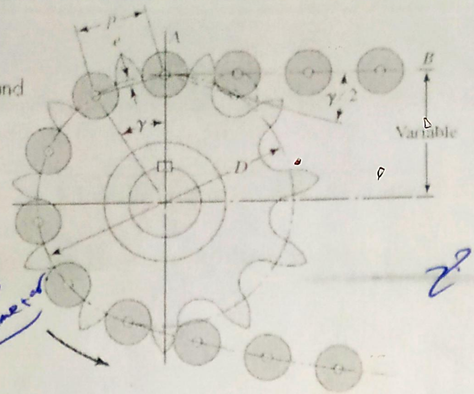
Figure 17-17

Engagement of a chain and sprocket.



P : chain pitch

D = sprocket pitch diameter

The angle $\gamma/2$, through which the link swings as it enters contact, is called the *angle of articulation*. $\frac{\gamma}{2}$

Pitch angle : γ

Figure 17–17 shows a sprocket driving a chain and rotating in a counterclockwise direction. Denoting the chain pitch by p, the pitch angle by γ, and the pitch diameter of the sprocket by D, from the trigonometry of the figure we see

$$\sin\frac{\gamma}{2} = \frac{p/2}{D/2} \quad \text{or} \quad D = \frac{p}{\sin(\gamma/2)} \qquad (a)$$

Since $\gamma = 360°/N$, where N is the number of sprocket teeth, Eq. (a) can be written

$$D = \frac{p}{\sin(180°/N)} \qquad (17\text{–}29)$$

it is important to reduce the angle of articulation as much as possible. Why?

Roller Chains continued

* when the sprocket has turned an angle of $\gamma/2$, the chain line AB is moving up and down, and that the lever arm varies with rotation through the pitch angle, this results in an uneven (not constant) chain exit velocity. Think of the sprocket as a polygon!! متفاوت

* The chain velocity V *is defined as the number of feet coming off the sprocket per unit time*. Thus the chain velocity in feet per minute is

$$V = \frac{Npn}{12} \qquad (17\text{–}30)$$

where N = number of sprocket teeth

p = chain pitch, in

n = sprocket speed, rev/min

* The maximum exit velocity of the chain is:

Max

$$v_{max} = \frac{\pi Dn}{12} = \frac{\pi np}{12\sin(\gamma/2)} \qquad (b)$$

* The minimum exit velocity occurs at a diameter *d, smaller than D.* Using the geometry of Fig. 17–17, we find $\boxed{d = D\cos\frac{\gamma}{2}}$, and the *minimum exit* velocity is:

$$v_{min} = \frac{\pi dn}{12} = \frac{\pi np}{12}\frac{\cos(\gamma/2)}{\sin(\gamma/2)} \qquad (d)$$

min

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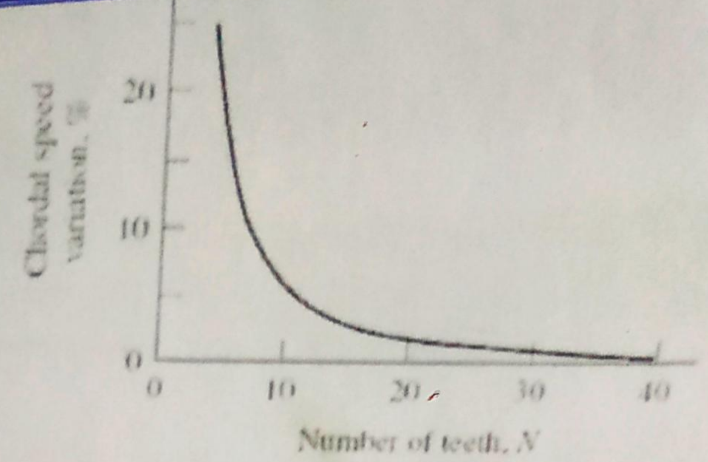
Now substituting $\gamma/2 = 180°/N$ and employing Eqs. (17–30), (b), and (d), we find the speed variation to be

$$\frac{\Delta V}{V} = \frac{V_{max} - V_{min}}{V} = \frac{\pi}{N}\left[\frac{1}{\sin(180°/N)} - \frac{1}{\tan(180°/N)}\right] \qquad (17\text{–}31)$$

This is called the *chordal speed variation* and is plotted in Fig. 17–18.

%

Figure 17–18



* For smooth operation at moderate and high speeds it is considered good practice to use a driving sprocket with at least 17 teeth; 19 or 21 will, of course, give a better life expectancy with less chain noise.

N

* What is the effect of high chordal speed variation on the system?

* Note: the most successful drives have velocity ratios up to 6:1

Roller Chains continued

Sprocket Speed, rev/min	25	35	40	41	50	60
			ANSI Chain Number			
50	0.05	0.16	0.37	0.20	0.72	1.24
100	0.09	0.29	0.69	0.38	1.34	2.31
150	0.13*	0.41*	0.99*	0.55*	1.92*	3.32
200	0.16*	0.54*	1.29	0.71	2.50	4.30
300	0.23	0.78	1.85	1.02	3.61	6.20
400	0.30*	1.01*	2.40	1.32	4.67	8.03
500	0.37	1.24	2.93	1.61	5.71	9.81
600	0.44*	1.46*	3.45*	1.90*	6.72*	11.6
700	0.50	1.68	3.97	2.18	7.73	13.3
800	0.56*	1.89*	4.48*	2.46*	8.71*	15.0
900	0.62	2.10	4.98	2.74	9.69	16.7
1000	0.68*	2.31*	5.48	3.01	10.7	18.3
1200	0.81	2.73	6.45	3.29	12.6	21.6
1400	0.93*	3.13*	7.41	2.61	14.4	18.1
1600	1.05*	3.53*	8.36	2.14	12.8	14.8
1800	1.16	3.93	8.96	1.79	10.7	12.4
2000	1.27*	4.32*	7.72*	1.52*	9.23*	10.6
2500	1.56	5.28	5.51*	1.10*	6.58*	7.57
3000	1.84	5.64	4.17	0.83	4.98	5.76
Type A		Type B			Type C	

*Estimated from ANSI tables by linear interpolation.

Note: Type A—manual or drip lubrication, type B—bath or disk lubrication, type C—oil-stream lubrication.

Roller Chains continued

Sprocket Speed, rev/min		80	100	120	140	160	180	200	240
					ANSI Chain Number				
50	Type A	2.88	5.52	9.33	14.4	20.9	28.9	38.4	61.8
100		5.38	10.3	17.4	26.9	39.1	54.0	71.6	115
150		7.75	14.8	25.1	38.8	56.3	77.7	103	166
200		10.0	19.2	32.5	50.3	72.9	101	134	215
300		14.5	27.7	46.8	72.4	105	145	193	310
400		18.7	35.9	60.6	93.8	136	188	249	359
500	Type B	22.9	43.9	74.1	115	166	204	222	0
600		27.0	51.7	87.3	127	141	155	169	
700		31.0	59.4	89.0	101	112	123	0	
800		35.0	63.0	72.8	82.4	91.7	101		
900		39.9	52.8	61.0	69.1	76.8	84.4		
1000		37.7	45.0	52.1	59.0	65.6	72.1		
1200		28.7	34.3	39.6	44.9	49.9	0		
1400		22.7	27.2	31.5	35.6	0			
1600		18.6	22.3	25.8	0				
1800		15.6	18.7	21.6					
2000		13.3	15.9	0					
2500		9.56	0.40						
3000		7.25	0						
Type C				Type C′					

Note:
numbe
inches,
series,
exampl

Table 17-21 Single-Strand Sprocket Tooth Counts Available from One Supplier*

No.	Available Sprocket Tooth Counts
25	8-30, 32, 34, 35, 36, 40, 42, 45, 48, 54, 60, 64, 65, 70, 72, 76, 80, 84, 90, 95, 96, 102, 112, 120
35	4-45, 48, 52, 54, 60, 64, 65, 68, 70, 72, 76, 80, 84, 90, 95, 96, 102, 112, 120
41	6-60, 64, 65, 68, 70, 72, 76, 80, 84, 90, 95, 96, 102, 112, 120
40	8-60, 64, 65, 68, 70, 72, 76, 80, 84, 90, 95, 96, 102, 112, 120
50	8-60, 64, 65, 68, 70, 72, 76, 80, 84, 90, 95, 96, 102, 112, 120
60	8-60, 62, 63, 64, 65, 66, 67, 68, 70, 72, 76, 80, 84, 90, 95, 96, 102, 112, 120
80	8-60, 64, 65, 68, 70, 72, 76, 78, 80, 84, 90, 95, 96, 102, 112, 120
100	8-60, 64, 65, 67, 68, 70, 72, 74, 76, 80, 84, 90, 95, 96, 102, 112, 120
120	9-45, 46, 48, 50, 52, 54, 55, 57, 60, 64, 65, 67, 68, 70, 72, 76, 80, 84, 90, 96, 102, 112, 120
140	9-28, 30, 31, 32, 33, 34, 35, 36, 37, 39, 40, 42, 43, 45, 48, 54, 60, 64, 65, 68, 70, 72, 76, 80, 84, 96
160	8-30, 32-36, 38, 40, 45, 46, 50, 52, 53, 54, 56, 57, 60, 62, 63, 64, 65, 66, 68, 70, 72, 73, 80, 84, 96
180	13-25, 28, 35, 39, 40, 45, 54, 60
200	9-30, 32, 33, 35, 36, 39, 40, 42, 44, 45, 48, 50, 51, 54, 56, 58, 59, 60, 63, 64, 65, 68, 70, 72
240	9-30, 32, 35, 36, 40, 44, 45, 48, 52, 54, 60

*Morse Chain Company, Ithaca, NY. Type B hub sprockets.

Roller Chains continued

Table 17-22

Tooth Correction Factors, K_1

K_1

Table 17-23

Multiple-Strand Factors, K_2

K_2

* The fatigue strength of link plates governs capacity at lower speeds. The American Chain Association (ACA) gives, for single-strand chain, the nominal power H_1 , *link-plate limited*, as

$$H_1 = 0.004 N_1^{1.08} n_1^{0.9} p^{(3-0.07p)} \quad \text{hp} \tag{17-32}$$

and the nominal power H_2, *roller-limited, as*

$$H_2 = \frac{1000 K_r N_1^{1.5} p^{0.8}}{n_1^{1.5}} \quad \text{hp} \tag{17-33}$$

where N_1 = number of teeth in the smaller sprocket
n_1 = sprocket speed, rev/min
p = pitch of the chain, in
K_r = 29 for chain numbers 25, 35; 3.4 for chain 41; and 17 for chains 40–240

The constant 0.004 becomes 0.0022 for no. 41 lightweight chain. The tabulated horsepower in Table 17–20 is $H_{nom} = \min(H_1, H_2)$.

Roller Chains continued

It is preferable to have an odd number of teeth on the driving sprocket (17, 19, . . .) and an even number of pitches in the chain to avoid a special link. The approximate length of the chain L in pitches is

$$\frac{L}{p} \doteq \frac{2C}{p} + \frac{N_1 + N_2}{2} + \frac{(N_2 - N_1)^2}{4\pi^2 C/p} \tag{17-34}$$

The center-to-center distance C is given by

$$C = \frac{p}{4}\left[-A + \sqrt{A^2 - 8\left(\frac{N_2 - N_1}{2\pi}\right)^2}\right] \tag{17-35}$$

where

$$A = \frac{N_1 + N_2}{2} - \frac{L}{p} \tag{17-36}$$

The allowable power H_a is given by

$$\boxed{H_a = K_1 K_2 H_{tab}} \tag{17-37}$$

where K_1 = correction factor for tooth number other than 17 (Table 17–22)
K_2 = strand correction (Table 17–23)

The horsepower that must be transmitted H_d is given by

$$H_d = H_{nom} K_s n_d \tag{17-38}$$

Equation (17–32) is the basis of the pre-extreme power entries (vertical entri
Table 17–20, and the chain power is limited by link-plate fatigue. Equation (17–3
the basis for the post-extreme power entries of these tables, and the chain power per-
formance is limited by impact fatigue. The entries are for chains of 100 pitch length and
17-tooth sprocket. For a deviation from this

$$H_2 = 1000\left[K_r\left(\frac{N_1}{n_1}\right)^{1.5} p^{0.8}\left(\frac{L_p}{100}\right)^{0.4}\left(\frac{15\,000}{h}\right)^{0.4}\right] \qquad (17\text{–}39)$$

where L_p is the chain length in pitches and h is the chain life in hours

Eq. (17–39) can be written as a trade-off equation in the following form:

$$\frac{H_2^{2.5}h}{N_1^{3.75}L_p} = \text{constant} \qquad (17\text{–}40)$$

حذف

If tooth-correction factor K_1 is used, then omit the term $N_1^{3.75}$

The maximum speed (rev/min) for a chain drive is limited by

$$n_1 \le 1000\left[\frac{82.5}{7.95^p(1.0278)^{N_1}(1.323)^{F/1000}}\right]^{1/(1.59\log p+1.873)} \quad \text{rev/min}$$

where F is the chain tension in pounds.

Solution of Problem 17-24

Equate Eqs. (17–32) and (17–33) to find the rotating speed n_1 at which the power equates and marks the division between the premaximum and the postmaximum power domains.

(*a*) Show that

$$n_1 = \left[\frac{0.25(10^6)K_r N_1^{0.42}}{p^{(2.2-0.07p)}}\right]^{1/2.4}$$

(*b*) Find the speed n_1 for a no. 60 chain, $p = 0.75$ in, $N_1 = 17$, $K_r = 17$, and confirm from Table 17–20.

Solution ➡ **(a)** Eq. (17-32): $\quad H_1 = 0.004N_1^{1.08}n_1^{0.9}p^{(3-0.07p)}$

Eq. (17-33): $\quad H_2 = \dfrac{1000K_r N_1^{1.5}p^{0.8}}{n_1^{1.5}}$

Equating and solving for n_1 gives

$$n_1 = \left[\frac{0.25(10^6)K_r N_1^{0.42}}{p^{(2.2-0.07p)}}\right]^{1/2.4} \quad \textit{Ans.}$$

(b) For a No. 60 chain, $p = 6/8 = 0.75$ in, $N_1 = 17$, $K_r = 17$

$$n_1 = \left\{\frac{0.25(10^6)(17)(17)^{0.42}}{0.75^{[2.2-0.07(0.75)]}}\right\}^{1/2.4} = 1227 \text{ rev/min} \quad \textit{Ans.}$$

Table 17-20 confirms that this point occurs at 1200 ± 200 rev/min.

17-25 A double-strand no. 60 roller chain is used to transmit power between a 13-tooth driving sprocket rotating at 300 rev/min and a 52-tooth driven sprocket.

(*a*) What is the allowable horsepower of this drive?

(*b*) Estimate the center-to-center distance if the chain length is 82 pitches.

(*c*) Estimate the torque and bending force on the driving shaft by the chain if the actual horsepower transmitted is 30 percent less than the corrected (allowable) power.

Solution

Given: a double strand No. 60 roller chain with $p = 0.75$ in, $N_1 = 13$ teeth at 300 rev/min $N_2 = 52$ teeth.

(a) Table 17-20: $H_{tab} = 6.20$ hp

Table 17-22: $K_1 = 0.75$

Table 17-23: $K_2 = 1.7$

Use $K_s = 1$

Eq. (17-37):

$$H_a = K_1 K_2 H_{tab} = 0.75(1.7)(6.20) = 7.91 \text{ hp} \quad Ans.$$

constant = $\dfrac{(H\ \text{tab old})(15000)^{2.5}}{L/p}$

$b = \left[\dfrac{\text{Constant}\ (L/p)}{20\ 000}\right]^{1/2.5}$

Solution of Problem 17-25 continued

(b) Eqs. (17-35) and (17-36) with $L/p = 82$

$$A = \frac{13 + 52}{2} - 82 = -49.5$$

$$C = \frac{p}{4}\left[49.5 + \sqrt{49.5^2 - 8\left(\frac{52-13}{2\pi}\right)^2}\right] = 23.95p$$

$C = 23.95(0.75) = 17.96$ in, round up to 18 in *Ans.*

(c) For 30 percent less power transmission,

$$H = 0.7(7.91) = 5.54 \text{ hp}$$

$$T = \frac{63025(5.54)}{300} = 1164 \text{ lbf} \cdot \text{in} \quad Ans.$$

$T = \dfrac{63025\,H_d}{n}$

Eq. (17-29):

Solution of Problem 17-27 continued

Table 17-22: $K_1 = 1$

Table 17-23: $K_2 = 1, 1.7, 2.5, 3.3$ for 1 through 4 strands

$$H'_{tab} = \frac{1.5(1.1)(25)}{(1)K_2} = \frac{41.25}{K_2}$$

Prepare a table to help with the design decisions:

Strands	K_2	H'_{tab}	Chain No.	H_{tab}	n_{fs}	Lub. Type
1	1.0	41.3	100	59.4	1.58	B
2	1.7	24.3	80	31.0	1.40	B
3	2.5	16.5	80	31.0	2.07	B
4	3.3	12.5	60	13.3	1.17	B

Design Decisions

We need a figure of merit to help with the choice. If the best was 4 strands of No. 60 chain, then

Decision #1 and #2: Choose four strand No. 60 roller chain with $n_{fs} = 1.17$.

$$n_{fs} = \frac{K_1 K_2 H_{tab}}{K_s H_{nom}} = \frac{1(3.3)(13.3)}{1.5(25)} = 1.17$$

Decision #3: Choose Type B lubrication

Analysis:

Table 17-20: $H_{tab} = 13.3$ hp

Table 17-19: $p = 0.75$ in

Try $C = 30$ in in Eq. (17-34):

$$\frac{L}{p} = \frac{2C}{p} + \frac{N_1 + N_2}{2} + \frac{(N_2 - N_1)^2}{4\pi^2 C/p}$$

$$= 2(30/0.75) + \frac{17 + 84}{2} + \frac{(84 - 17)^2}{4\pi^2(30/0.75)}$$

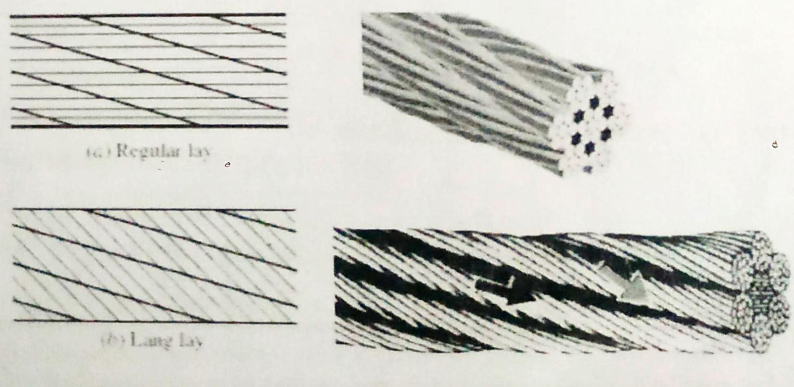
$$= 133.3 \rightarrow 134$$

From Eq. (17-35) with $p = 0.75$ in, $C = 30.26$ in.

Wire Ropes

* Wire ropes are made out of steel wires and are used in many applications (such as hoisting, haulage, marine, etc...).

- There are two basic ways of winding of wire ropes:
  - *Regular lay*: wires and strands are twisted in opposite directions (*do not kink or untwist*). (Fig. 17-19-a)
  - *Lang lay*: wires and strands are twisted in the same direction (*more resistance to wear and fatigue*). (Fig. 17-19-b)



(a) Regular lay

(b) Lang lay

Figure 17-19

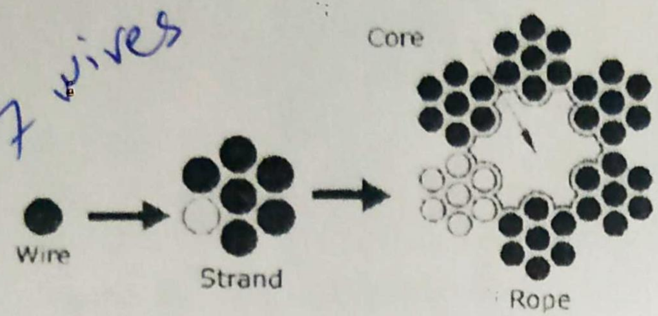
Types of wire rope; both lays are available in either right or left hand

d

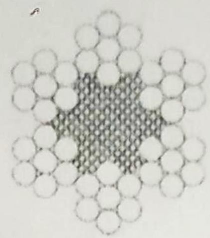
- Ropes are designated by <u>size and configuration</u>, for example: *25-mm 6x7* haulage rope means: diameter is *25 mm* and has *6* strands each having 7 wires.
  - ❖ <u>*Table 17-24*</u> lists some of the standard ropes along with their properties. Also see <u>*Table 17-27*</u>.

6 strands

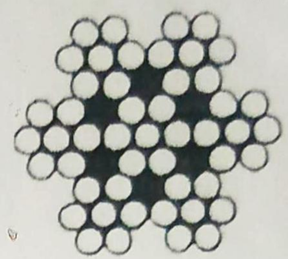
7 wires



Parts of Wire Rope



6 × 7 rope



7x7 rope

Table 17–27

Some Useful Properties of 6 × 7, 6 × 19, and 6 × 37 Wire Ropes

Wire Rope	Weight per Foot w, lbf/ft	Weight per Foot Including Core w, lbf/ft	Minimum Sheave Diameter D, in	Better Sheave Diameter D, in	Diameter of Wires d_w, in	Area of Metal A_m, in²	Rope Young's Modulus E_r, psi
6 × 7	$1.50d^2$		$42d$	$72d$	$0.111d$	$0.38d^2$	13×10^6
6 × 19	$1.60d^2$	$1.76d^2$	$30d$	$45d$	$0.067d$	$0.40d^2$	12×10^6
6 × 37	$1.55d^2$	$1.71d^2$	$18d$	$27d$	$0.048d$	$0.40d^2$	12×10^6

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Wire Ropes continued

- When a rope passes around a sheave, bending stress develops (*especially in the outer wires*) due to flexing.
  - Using mechanics principles, the stress in one of the wires of the rope can be found as:

Wire stress.

stress on one $\boxed{\text{wire}}$

$$\sigma = E_r \frac{d_{wire}}{D}$$

where,

E_r: modules of elasticity of the rope

d_w: diameter of the wire

D: sheave diameter

✓ This equation shows the importance of using large diameter sheaves (where it reduces the stress developed in the outer wires).

✓ The recommended D/d_w ratio is 400 and up.

- Tension in the rope causing the same stress caused by bending is called "*the equivalent bending load*", F_b, and it is found as:

F b

$$\boxed{F_b = \sigma A_m = \frac{E_r d_w A_m}{D}}$$

Tension in rope causing the same stress caused by bending

where,

A_m: is the area of the metal in the rope, and it is usually $A_m = 0.38d^2$ (or from *Table 17-27*)

d_w: diameter of the wire & D: sheave diameter

- Wire ropes are selected according to two considerations;
  - Static considerations: the ability of the rope to carry the loads.
  - Wear life (*fatigue*) considerations: the ability of the rope to live for a certain number of load cycles.

Wire Ropes continued

نقارن الـ Tension الناتج عن الـ load بـ ultimate tensile load لـ تبع الـ rope

- The tension due to loads is then compared to the ultimate tensile load of the rope to find the static factor of safety.

$$\boxed{F_u = \textit{strength of the rope} \times \textit{nominal area of the rope}}$$

Maximum load that can be supported

Thus, the static factor of safety is:

$$\boxed{n_s = \frac{F_u}{F_t}}$$

➤ However, the ultimate tensile load F_u must be reduced due to the increased tension caused by flexing the rope over the sheave and thus the factor of safety can be found as:

$$\boxed{n_s = \frac{F_u - F_b}{F_t}}$$

more accurate

❖ *Table 17-25* gives the minimum rope factors of safety for different applications.

Table 17-25

Minimum Factors of Safety for Wire Rope*

Source: Compiled from a variety of sources, including ANSI A17.1-1978.

Track cables	3.2	Passenger elevators, ft/min:	
Guys	3.5	50	7.60
Mine shafts, ft:		300	9.20
Up to 500	8.0	800	11.25
1000–2000	7.0	1200	11.80
2000–3000	6.0	1500	11.90
Over 3000	5.0	Freight elevators, ft/min:	
Hoisting	5.0	50	6.65
Haulage	6.0	300	8.20
Cranes and derricks	6.0	800	10.00
Electric hoists	7.0	1200	10.50
Hand elevators	5.0	1500	10.55
Private elevators	7.5	Powered dumbwaiters, ft/min:	
Hand dumbwaiter	4.5	50	4.8
Grain elevators	7.5	300	6.6
		500	8.0

*Use of these factors does not preclude a fatigue failure.

Fatigue considerations:

1

- The amount of wear that occurs in ropes depends on the bearing pressure on the rope caused by the sheave and by the number of bends (*number of the passes of the rope over the sheave*) of the rope during operation.

2

- The allowable fatigue tension (*fatigue strength*) for a rope is found as:

$$\boxed{F_f = \frac{(P/S_u)S_u dD}{2}}$$

Maximum tension that can be supported under certain bearing pressure (P) for a certain number of bends.

Where,

(P/S_u): bearing pressure to ultimate strength ratio. It is found according to the specified life from Fig 17-21.

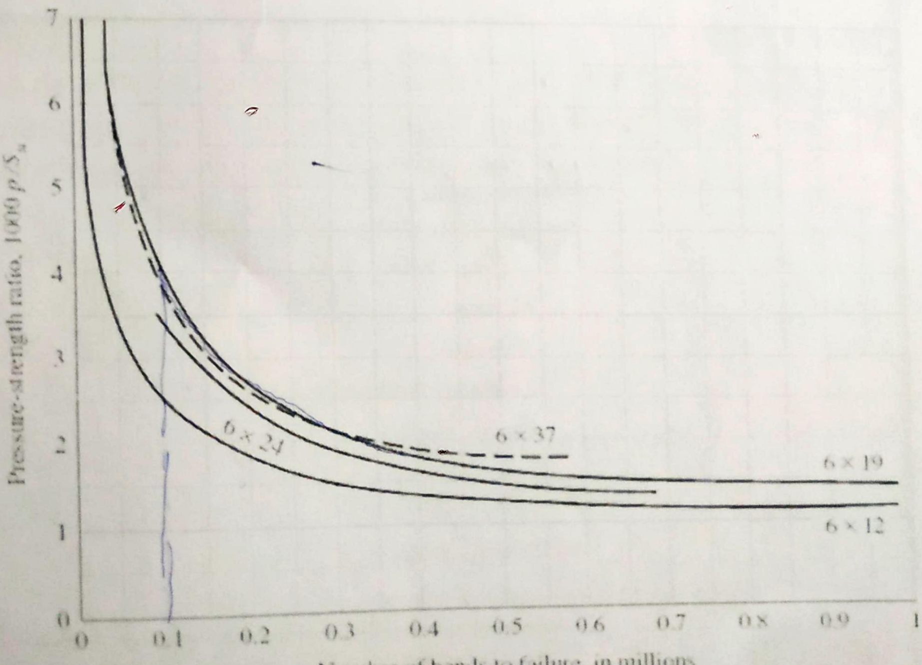
S_u: ultimate tensile strength of the wires.

d: diameter of the rope & D: diameter of the sheave.

Wire Ropes continued

Figure 17–21

Experimentally determined relation between the fatigue life of wire rope and the sheave pressure.



- It should be noted that S_u is the ultimate strength of the wires not the strength of the rope. (*it usually not listed in the tables but it can be determined from a hardness test*).*

- Thus, the fatigue factor of safety can be found as:

$$\boxed{n_f = \frac{F_f - F_b}{F_t}}$$

- It should be understood that the fatigue failure in wire ropes is not sudden, as in solid bodies, but rather progressive. It shows as breaking of the outside wires (since they are subjected to highest stress). Therefore it can be detected by periodic inspection.

* For *steels*, the relationship between the minimum ultimate strength and the Brinell hardness number for $200 \le H_B \le 450$ is found to be

$$S_u = \begin{cases} 0.495 H_B & \text{kpsi} \\ 3.41 H_B & \text{MPa} \end{cases} \qquad (2\text{–}17)$$

* Some guidance in strength of individual wirers is

Improved plow steel (monitor)	$240 < S_u < 280$ kpsi
Plow steel	$210 < S_u < 240$ kpsi
Mild plow steel	$180 < S_u < 210$ kpsi

Example 17-6

Given a 6 × 19 monitor steel ($S_u = 240$ kpsi) wire rope.
(*a*) Develop the expressions for rope tension F_t, fatigue tension F_f, equivalent bending tensions F_b, and fatigue factor of safety n_f for a 531.5-ft, 1-ton cage-and-load mine hoist with a starting acceleration of 2 ft/s² as depicted in Fig. 17–23. The sheave diameter is 72 in.
(*b*) Using the expressions developed in part (*a*), examine the variation in factor of safety n_f for various wire rope diameters d and number of supporting ropes m.

Solution

(*a*) Rope tension F_t from Eq. (17–46) is given by

$$F_t = \left(\frac{W}{m} + wl\right)\left(1 + \frac{a}{g}\right) = \left[\frac{2000}{m} + 1.60d^2(531.5)\right]\left(1 + \frac{2}{32.2}\right)$$

$$= \frac{2124}{m} + 903d^2 \text{ lbf}$$

- It should be noted that S_u is the ultimate strength of the wires not the strength of the rope. (*it usually not listed in the tables but it can be determined from a hardness test*).*

- Thus, the fatigue factor of safety can be found as:

$$\boxed{n_f = \frac{F_f - F_b}{F_t}}$$

- It should be understood that the fatigue failure in wire ropes is not sudden, as in solid bodies, but rather progressive. It shows as breaking of the outside wires (since they are subjected to highest stress). Therefore it can be detected by periodic inspection.

* For *steels*, the relationship between the minimum ultimate strength and the Brinell hardness number for $200 \le H_B \le 450$ is found to be

$$S_u = \begin{cases} 0.495 H_B & \text{kpsi} \\ 3.41 H_B & \text{MPa} \end{cases} \qquad (2\text{–}17)$$

* Some guidance in strength of individual wirers is

Improved plow steel (monitor)	$240 < S_u < 280$ kpsi
Plow steel	$210 < S_u < 240$ kpsi
Mild plow steel	$180 < S_u < 210$ kpsi

Example 17-6

Given a 6 × 19 monitor steel ($S_u = 240$ kpsi) wire rope.

(*a*) Develop the expressions for rope tension F_t, fatigue tension F_f, equivalent bending tensions F_b, and fatigue factor of safety n_f for a 531.5-ft, 1-ton cage-and-load mine hoist with a starting acceleration of 2 ft/s² as depicted in Fig. 17–23. The sheave diameter is 72 in.

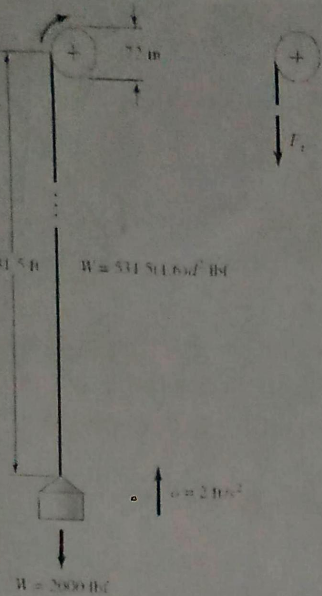
(*b*) Using the expressions developed in part (*a*), examine the variation in factor of safety n_f for various wire rope diameters d and number of supporting ropes m.

Solution

(*a*) Rope tension F_t from Eq. (17–46) is given by

$$F_t = \left(\frac{W}{m} + wl\right)\left(1 + \frac{a}{g}\right) = \left[\frac{2000}{m} + 1.60d^2(531.5)\right]\left(1 + \frac{2}{32.2}\right)$$

$$= \frac{2124}{m} + 903d^2 \text{ lbf}$$



From Fig. 17–21, use $p/S_u = 0.0014$. Fatigue tension F_f from Eq. (17–47) is given by

$$F_f = \frac{(p/S_u)S_u D d}{2} = \frac{0.0014(240\,000)72d}{2} = 12\,096d \text{ lbf}$$

Equivalent bending tension F_b from Eq. (17–48) and Table 17–27 is given by

$$F_b = \frac{E_r d_w A_m}{D} = \frac{12(10^6)0.067d(0.40d^2)}{72} = 4467d^3 \text{ lbf}$$

Factor of safety n_f in fatigue from Eq. (17–50) is given by

$$n_f = \frac{F_f - F_b}{F_t} = \frac{12\,096d - 4467d^3}{2124/m + 903d^2}$$

Example 17-6 continued

(b) Form a table as follows:

	n_f			
d	**m = 1**	**m = 2**	**m = 3**	**m = 4**
0.25	1.355	2.641	3.865	5.029
0.375	1.910	3.617	5.150	6.536
0.500	2.336	4.263	5.879	7.254
0.625	2.612	4.573	6.099	7.331
0.750	2.731	4.578	5.911	6.918
0.875	2.696	4.330	5.425	6.210
1.000	2.520	3.882	4.736	5.320

Wire rope sizes are discrete, as is the number of supporting ropes. Note that for each m the factor of safety exhibits a maximum. Predictably the largest factor of safety increases with m. If the required factor of safety were to be 6, only three or four ropes could meet the requirement. The sizes are different: $\frac{5}{8}$-in ropes with three ropes or $\frac{3}{8}$-in ropes with four ropes. The costs include not only the wires, but the grooved winch drums.

A mine hoist uses a 2-in 6 × 19 monitor-steel wire rope. The rope is used to haul loads ... from the shaft 480 ft deep. The drum has a diameter of 6 ft. the sheaves are of good-quality cast steel, and the smallest is 3 ft in diameter.

(*a*) Using a maximum hoisting speed of 1200 ft/min and a maximum acceleration of 2 ft/s², estimate the stresses in the rope.

(*b*) Estimate the various factors of safety.

Solution

(a) Monitor steel 2-in 6 × 19 rope. 480 ft long , sheave diameter = 3x12 = 36 in.

the ultimate strength of the wire S_u is = 240 kpsi. W = 4 tons = 4 X 2000 = 8000 lb

Table 17-24 gives the nominal tensile strength for the wire rope as 106 kpsi.

Therefore, The ultimate load is

$$F_u = (S_u)_{\text{nom}} A_{\text{nom}} = 106\left[\frac{\pi(2)^2}{4}\right] = 333 \text{ kip}$$

rope

A

10.036

Solution of Problem 17-29 continued

The tensile loading of the wire is given by Eq. (17-46)

$$F_t = \left(\frac{W}{m} + wl\right)\left(1 + \frac{a}{g}\right)$$

KPsi

W = 8 kip. m = 1

Kilo pound/squ inc

Table (17-24):

$$wl = 1.60d^2l = 1.60(2^2)(480) = 3072 \text{ lbf} \quad \text{or} \quad 3.072 \text{ kip}$$

Therefore,

$$F_t = (8 + 3.072)\left(1 + \frac{2}{32.2}\right) = 11.76 \text{ kip} \quad Ans.$$

Eq. (17-48):

$$F_b = \frac{E_r d_w A_m}{D}$$

Psi

$Er_{Psi} \times 10^{-3}$

$\boxed{= Er\ KPsi}$

and for the 72-in drum

Er

0.38 d^2

$$F_b = \frac{12(10^6)(2/13)(0.38)(2^2)(10^{-3})}{\ldots} = 78 \text{ kip}$$

(b) Factors of safety

Static, no bending:

$$n = \frac{F_u}{F_i} = \frac{333}{11.76} = 28.3 \quad \textit{Ans.}$$

Static, with bending:

Eq. (17-49):
$$n_s = \frac{F_u - F_b}{F_i} = \frac{333 - \boxed{78}}{11.76} = \boxed{21.7} \quad \textit{Ans.}$$

* To find the fatigue factors of safety, we need first to find F_f , we shall assume the life to be 1 million cycle; so using Eq. (17-44) and Fig. 17-21, we get:

$$(p/S_u) = 0.0014$$

$$S_u = 240 \text{ kpsi.} \quad \text{p. 908}$$

$$F_f = \frac{0.0014(240)(2)\,\boxed{(36)}}{2} = \underline{12.1}\ \text{kip}$$

Solution of Problem 17-29 continued

Comments:

- There are a number of factors of safety used in wire rope analysis. They are different, with different meanings. There is no substitute for knowing exactly which factor of safety is written
- Static performance of a rope in tension is impressive.
- In this problem, at the sheave we have a finite life.
- The remedy for fatigue is the use of smaller diameter ropes, with multiple ropes supporting the load. See Ex. 17-6 for the effectiveness of this approach.